

ANALYSIS OF THE RUNOFF VOLUMES OF THE WAVE BELONGS
TO MAXIMUM ANNUAL DISCHARGES

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Several hypothesis claim that more extremes in climatic in hydrological phenomena are anticipated. In order to verify such hypotheses we described the annual flood risk volume analysis carried out in the Váh River in Slovakia. In the present study, the annual maximum runoff volumes with t -day durations ($t=2$ -, 5-, 10- and 20-days) were calculated for an 85-year series (1931–2015) of mean daily discharges and maximum annual discharges of Váh River: Liptovský Mikuláš gauge. In the next section, we estimated the total volumes of the wave belongs to maximum annual discharges. The T -year volume values were calculated using Log-Pearson type III distribution. Statistical method was used to clarify how the maximum and total volumes of the Váh River changed over the selected period (1931–2015).

KEY WORDS: Váh River, wave volume, Log-Pearson III probability distribution, T -year volume

Introduction

The basic need for dimensioning of flood protection structures are designed values of hydrological characteristics which could have disaster effect. Determination of design values for extreme floods with very low probability of occurrence, it means with a long return period (once every 1000 years) is a very difficult and complex process, coupled with great uncertainty. When developing plans and maps of flood threats, it is desirable to use various methods – starting with historical hydrology (mapping historical records), through statistical methods of calculating design values, to using mathematical modelling of extreme hydrological situations and regionalization methods. Solution of some water management tasks requires knowing not only maximum discharge but also the shape of the flood wave or at least its volume. The damage of the protective dam may not occur as a result of high water levels or discharges, but also, as a result of long-time high volumes – wetting, overspill e.g. connection of two flood waves at the confluence. The significance of the flood wave volume as an important hydrological characteristic was evident, e.g. during the flood in 1965 on the Danube River, when the protective dams ruptured due to the long occurrence of high water level, not because of its extreme value (Zatkalík, 1970; Hladný et al., 1970). The similar situation was in the spring of 1941 on the Morava River when the flood lasted more than 3 months and volume was almost 2 times larger than the volume of the flood wave in 1997 with almost identical culmination.

Analyzing temporal changes in maximum runoff volume series of the Danube River was investigated in Halmová et. al., (2008). Szolgay et al. (2012) dealt with the estimation of the flood wave volume, which corresponds to the maximum design discharge with an return period of $T=10\,000$ years. From foreign authors, e.g. Beard (1956) dealt with determining maximum volumes. Author used theoretical exceedance curves to calculate annual maximum volumes of varying exceedance probability considering the duration of the flow wave t .

In assessment of the climate change impacts on the river runoff regime (extremes, flood hydrographs and drought periods), it is expected that the increase of air temperature may cause (or already has caused) the increase of extreme discharges and flood volumes. It is necessary regularly to check the validity of the assumptions in order to have correct statistical results (IACWD, 1982). Significant changes in the river basins (as urbanization or construction of the flood protection structures) may have influence on the hydrological extremes and can corrupt the frequency analysis application. It is well known that in some small streams were floods with atypical ratio of the extreme flood wave volume to its culmination, and belonged to the phenomena, the occurrence of which no one expected. Therefore, for the engineering praxis is necessary to study the flood wave volumes in time. In applied hydrology the problem is the assignment of flood wave volumes with a certain probability of occurrence to the corresponding T -year discharges.

The aim of this study is:

- assess the maximum annual runoff volumes V_{max}

lasting 2-, 5-, 10-, 20-days and total runoff volumes V of the wave belongs to annual maximum discharges of the Váh River: Liptovský Mikuláš (1931–2015);

- determine the theoretical exceedance probability curves;
- estimate the T -year annual maximum runoff volumes with t -day durations and total runoff volumes of the wave belongs to annual maximum discharge;
- analyze changes in the discharge wave volumes of the Váh River: Liptovský Mikuláš (1931–2015).

Case study area

The Váh River and Input data

The Váh River is an important and the longest Slovak river with a length of 403 km and a basin area of 19 696 km². It rises in the Tatra Mountains by the confluence of the White Váh and Black Váh (Fig. 1). The Váh River flows over northern and western Slovakia and finally feeds into the Danube near Komárno. The Váh River basin accounts for about 37% of water bearing of Slovakia. The Váh has a large number of tributaries, many of which are mountain streams from the Tatra Mountains and Carpathians (e.g. Belá, Orava, Kysuca, Rajčianka, Turiec, Malý Dunaj,...). Long-term daily discharges of the Váh River during the period of 1931–2015 reached value about of 20.4 m³ s⁻¹ at Liptovský Mikuláš gauge (basin drainage depth is 582.4 mm) and the maximum discharge reached value 540 m³ s⁻¹ (29th June 1958).

The course of annual peak discharges, long-term linear trend and 5-year moving trend are illustrated on the Figure 1. The annual peak discharges of the Váh River at Liptovský Mikuláš show decreasing long-term linear trend during the selected period of 1931–2015. The deviation of mean long-term annual discharge showed the driest period of 1986–1999 (Fig. 1). There were also occurred some extreme floods in 1934, 1948,

1958 or 1997 and relatively longer wet period in 1973–1981 (Fig. 1). The scenarios of changes of selected elements of the hydrosphere and biosphere in the Váh basin are reported in monography of Pekárová and Szolgay (2005) and in Jeneiova et al. (2014).

Methodology

To define the volumes of individual waves, we introduced the parameter t – runoff duration in days. In this way, we determined maximum runoff volumes of $t=2$ -, 5-, 10- and 20 days. The series of mean daily discharges were used to determine the annual maximum runoff volume V_{max} lasting 2-, 5-, 10- and 20- days. If the wave duration was less than 20 days, the steady discharges were included into the analysis. Figure 2 presents an example of the determination of maximum volumes with a given runoff duration.

For determination of the total duration and total volume of the wave, it was necessary to identify the beginning and end of the wave. It is quite difficult to identify the beginning and end of the discharge wave, in some cases. In our analysis, the beginning and end of the wave was determined approximately at the level of the long-term average daily discharge $\bar{Q}_d=21$ m³ s⁻¹ (1931–2015). We also assumed that there were no others significant atmospheric events.

In the world literature, there is a number of scientific papers dealing with the selection and testing of the suitability of theoretical probability distributions in estimating maximum values of hydrological characteristics (Cunnane 1989; Helsel and Hirsch, 2002; Langat et al., 2019). The type of statistical methods, especially selection of the theoretical probability distribution, which is used to estimate the extreme values, also influences the estimation of their return periods. Based on our knowledge, we propose to use only one type of distribution, namely the Log-Pearson distribution III. type (LPIII distribution). Log-Pearson distribution III. type is used to

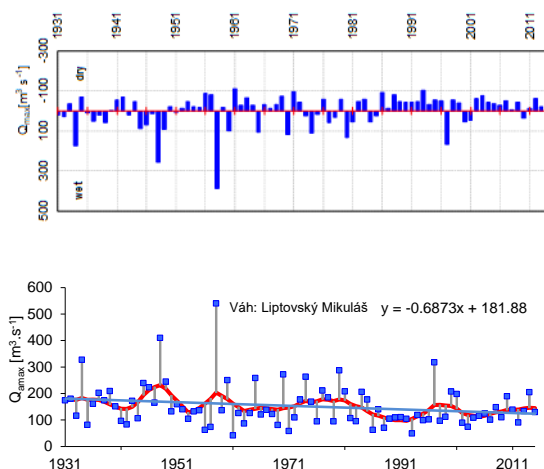
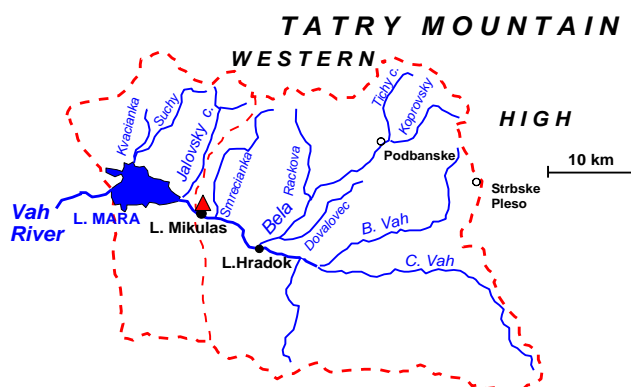


Fig. 1 The location of the selected Váh River section and right-up: deviation from long-term mean annual discharge during the period of 1931–2015 and right-down: maximum annual discharges, Váh: Liptovský Mikuláš (1931–2015), their linear trend and 5-year moving trend.

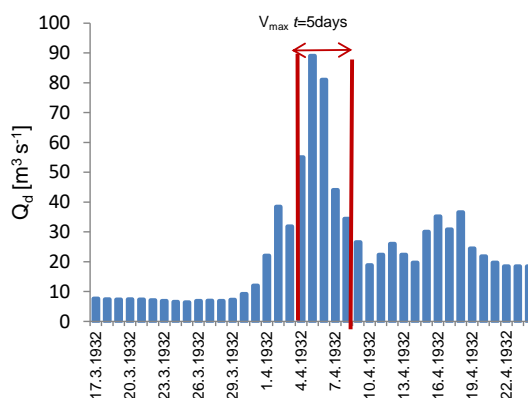


Fig. 2. Example of the determination of the maximum volume with a given runoff duration $t=5$ days on Váh River: Liptovský Mikuláš (1932).

estimate extremes in many natural processes and is one of the most commonly used probability distribution in hydrology (Phien and Jivajirajah, 1984; Pilon and Adamowski, 1993; Millington et al., 2011). The LPIII theoretical distribution belongs to the family of Pearson distributions, so called three parametric Gamma distributions, with logarithmic transformation of the data. This type of distribution is possible to proceed with regionalization of the LPIII distribution using the third parameter of this distribution – skew coefficient (asymmetry). The cumulative distribution function and probability distribution function according Hosking and Wallis (1997) are defined as:

If $\gamma \neq 0$ let $\alpha=4/\gamma^2$ and $\xi=\mu-2\sigma/\gamma$

If $\gamma > 0$ then:

$$F(x) = G(\alpha, \frac{x-\xi}{\beta})/\Gamma(\alpha) \quad (1)$$

$$f(x) = \frac{(x-\xi)^{\alpha-1} e^{-(x-\xi)/\beta}}{\beta^\alpha \Gamma(\alpha)} \quad (2)$$

where

ξ – location parameter;

α – shape parameter;

β – scale parameter;

Γ – Gamma function.

If $\gamma < 0$ then

$$F(x) = 1 - G(\alpha, \frac{x-\xi}{\beta})/\Gamma(\alpha) \quad (3)$$

$$f(x) = \frac{(\xi-x)^{\alpha-1} e^{-(\xi-x)/\beta}}{\beta^\alpha \Gamma(\alpha)} \quad (4)$$

The Kolmogorov-Smirnov test was performed to test the assumption that the discharge magnitudes follow the theoretical distributions. The p -value ($p \geq 0.05$) was used as a criterion for rejection of the proposed distribution hypothesis. The empirical probability curve

of the maximum volumes was calculated according equation (5):

$$P = \frac{m}{n+0,4} \quad (5)$$

where

m – variable order number – descending order to the statistical series;

n – number of variables.

The relationship between the probability of exceedance a given value in any year and its average return period T is (Szolgay et al., 1994):

$$p = 1 - e^{-1/T} \quad (6)$$

If $T \geq 10$ we can use simplified form of equation (6):

$$P = \frac{m}{T} \quad (7)$$

Results

Maximum annual runoff volumes V_{max} lasting 2-, 5-, 10-, 20-days

The annual maximum runoff volumes at a given runoff duration of the Váh River at Liptovský Mikuláš station during the period of 1931–2015 and their linear trends are presented in Figure 3. From the point of view of 2-days and 5-days annual maximum runoff volumes the highest values reached the flood in 1948 ($V_{max2}=39.1$ mil.m³ and $V_{max5}=71.4$ mil.m³). From the point of view of 10-days and 20-days annual maximum runoff volumes the highest values reached the flood in 1965 ($V_{max10}=106.2$ mil.m³ and $V_{max20}=175.2$ mil.m³). The maximal numbers of the annual peak discharges occurred in May. The maximum annual volumes show a slightly declining linear trend for 2-days and 5-days runoff duration. Figure 4 and Table 1 present T-year annual maximum runoff volumes of the Váh: Liptovský Mikuláš (Log-Pearson III).

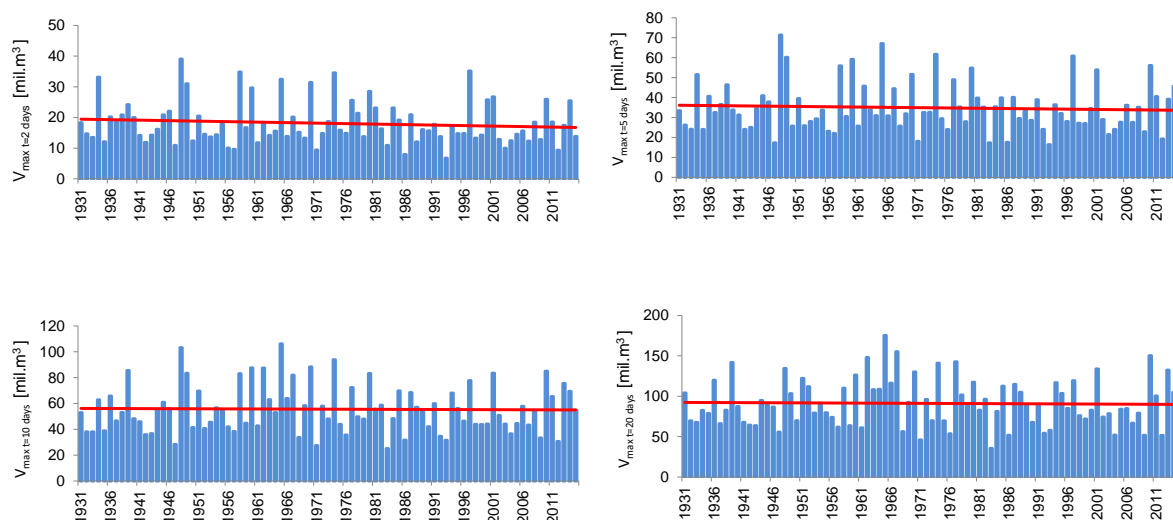


Fig. 3. Time course of annual maximum runoff volumes lasting 2-, 5-, 10-, 20-days, Váh River: Liptovský Mikuláš (1931–2015).

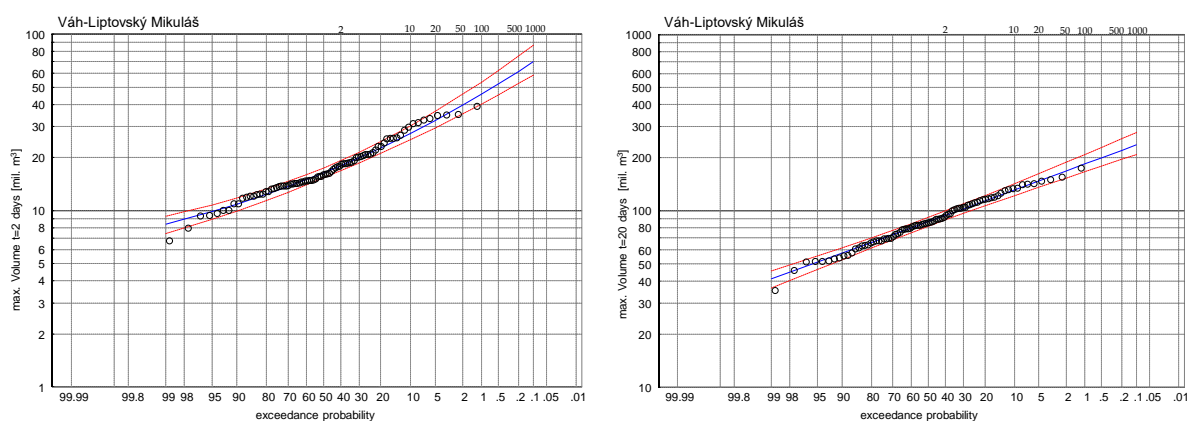


Fig. 4. The LPIII exceedance probability curve of the $V_{max,t}$ for a given runoff duration $t=2$ days (left) and $t=20$ days (right), Váh: Liptovský Mikuláš (1931–2015).

Table 1. T -year maximum discharges Q_{max} [$\text{m}^3 \text{s}^{-1}$] and T -year annual maximum runoff volumes $V_{max,t}$ [mil. m^3], Váh: Liptovský Mikuláš (1931–2015) (Log-Pearson III)

River: Gauging station	Q_T [$\text{m}^3 \text{s}^{-1}$]	$t=2$ days	$t=5$ days	$t=10$ days	$t=20$ days
Váh: Liptovský Mikuláš	Q_{50}	$V_{50max,t}$ [mil. m^3]			
	426	41	79	108	171
	Q_{100}	$V_{100max,t}$ [mil. m^3]			
	521	46	78	116	184
	Q_{500}	$V_{500max,t}$ [mil. m^3]			
	809	62	99	143	221
	Q_{1000}	$V_{1000max,t}$ [mil. m^3]			
	969	70	108	154	237

Maximum annual runoff volumes V_{max} lasting 2-, 5-, 10-, 20-days in two parts

With regard to the character of the upper part of the river Váh basin, where the seasonality of individual tributaries is manifested, we divided the data into two parts:

- I. December–May;
- II. June–November.

The maximum annual runoff volumes V_{max} lasting 2-, 5-, 10-, 20-days, show a constant or slightly decreasing linear trend for the I. part: December–May. The maximum

annual discharges and maximum annual volumes for a given runoff duration $t=2$ days and $t=20$ days for selected periods I. are presented in Figure 5. The maximum annual runoff volumes V_{max} lasting 2-days have decreasing linear trend during the II. part: June–November. For V_{max} lasting 5-, 10- and 20-days the trend is approached to constant value.

Calculated T -year maximum annual volumes V_{max} for runoff duration $t=2$ -days and 20-days calculated by LPII probability distribution for selected parts I. and II. are presented in Figure 6. T -year annual maximum runoff volumes V_{max} for selected parts are listed in Table 2.

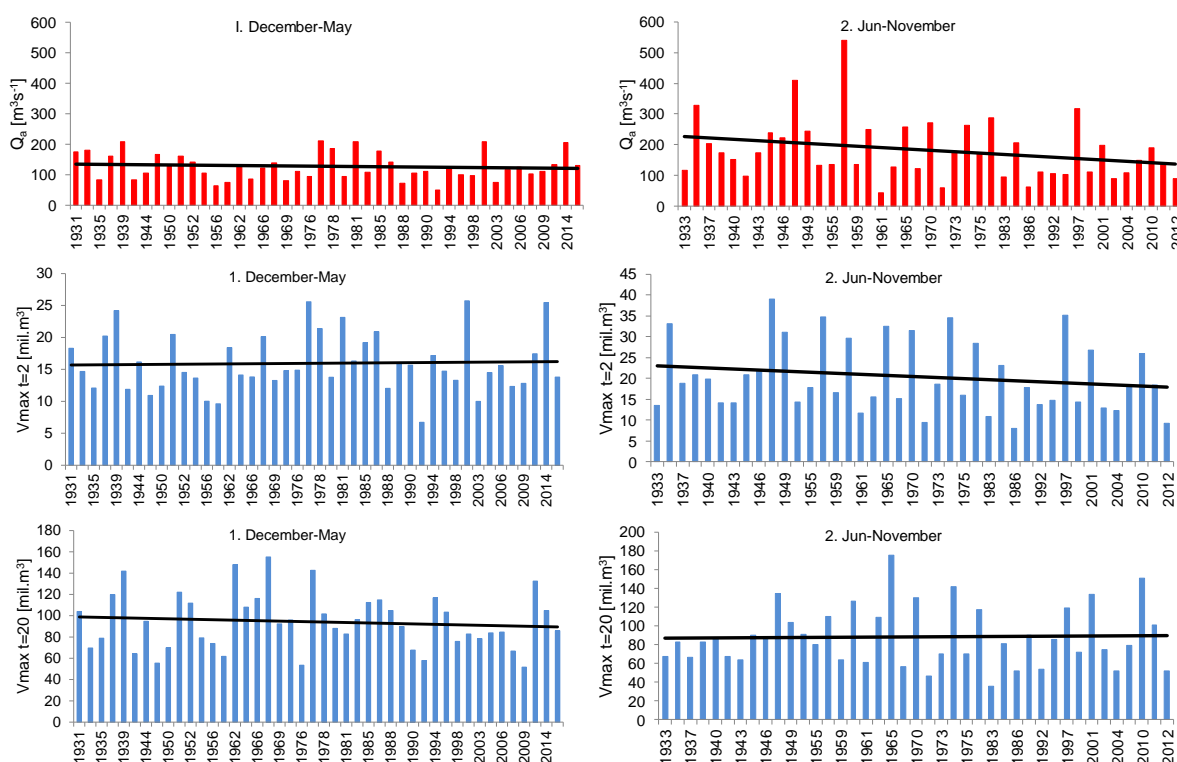


Fig. 5. The maximum annual discharges and maximum annual volumes for a given runoff duration $t=2$ days and $t=20$ days for selected parts I. and II., Váh: Liptovský Mikuláš (1931–2015).

Table 2. T -year annual maximum runoff volumes V_{max} [$mil. m^3$] for selected parts, Váh: Liptovský Mikuláš (1931–2015) (Log-Pearson III)

River: Gauging station	parts	$t=2$ days	$t=5$ days	$t=10$ days	$t=20$ days
Váh: Liptovský Mikuláš	V_{50max} [$mil. m^3$]				
	I.	27	51	91	160
	II.	44	80	117	172
	V_{100max} [$mil. m^3$]				
	I.	29	54	98	172
	II.	49	89	130	190
	V_{500max} [$mil. m^3$]				
	I.	33	60	114	200
	II.	63	112	161	232
	$V_{1000max}$ [$mil. m^3$]				
	I.	35	63	12	211
	II.	68	122	175	250

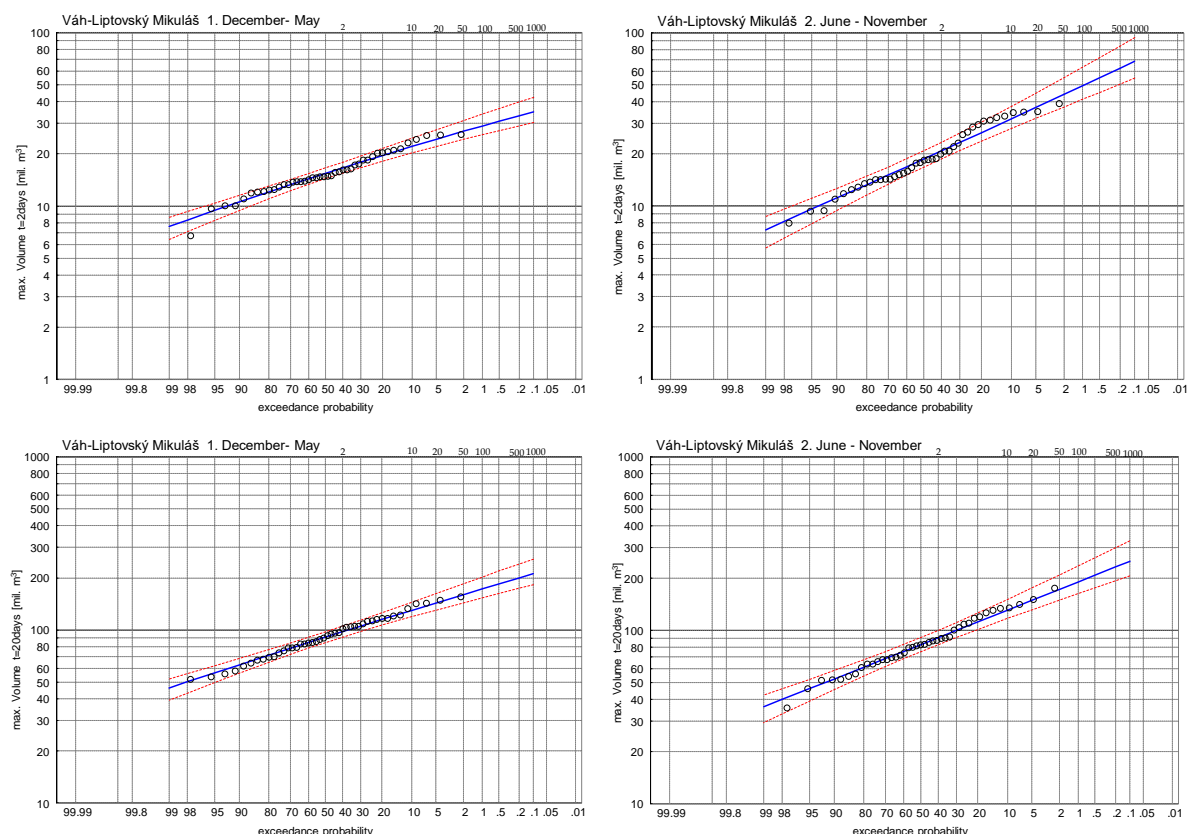


Fig. 6. The LPIII exceedance probability curve of the $V_{max,t}$ for a given runoff duration $t=2$ days and $t=20$ days for selected parts I. and II., Váh: Liptovský Mikuláš (1931–2015).

Total annual runoff volumes of the wave belongs total annual maximum discharge

As mentioned above in our analysis, the beginning and end of the wave was determined approximately at the level of the long-term average daily discharge $\bar{Q}_d=21 \text{ m}^3\text{s}^{-1}$ (1931–2015). Figure 7 illustrates the total runoff duration and month of annual maximum discharge occurrence.

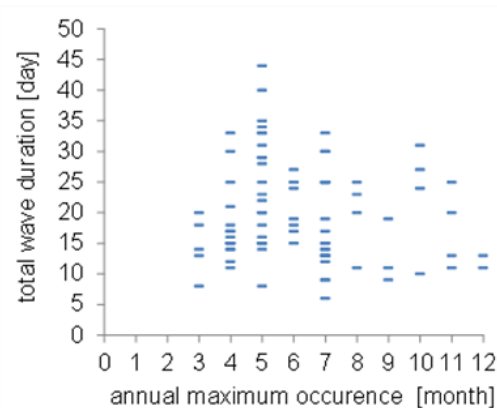


Fig. 7. Distribution of the total duration of the wave belongs to the annual discharges, Váh: Liptovský Mikuláš (1931–2015).

Figure 7 shows that the total wave durations above 25-days most often occur in May.

The mean total duration of the discharge waves with this limit was 20 days (Fig. 8a). The longest duration ($t=43$ days) with this criterion was identified for wave which occurred in April–May 2013 (Figure 8a). The maximum discharge of this wave was about $133.50 \text{ m}^3\text{s}^{-1}$. In contrast, the wave belongs to the highest annual maximum discharges (years 1948 and 1958) lasted only 25 days (Figure 8a). Calculated total volumes of the identified wave belongs to maximum annual discharges of the river Váh: Liptovský Mikuláš for the period 1931–2015 are illustrated in Figure 8b. The total volume and duration of the waves show a slightly increasing trend during the selected period (Fig. 8).

Based on calculated total runoff volumes of the wave belongs to annual maxima the T -year total volumes were calculated by Log-Pearson type III. probability distribution (Fig. 9). Table 3 listed T -year maximum discharges Q_{max} and T -year annual total runoff volumes belong to annual maximum discharges in Váh: Liptovský Mikuláš (1931–2015) (Log-Pearson III).

Total annual runoff volumes of the wave belongs to the annual maximum discharge in two parts

The total runoff volumes V and total runoff duration t of the waves show a markedly increasing linear trend for

the I. period: December–May (Fig. 10). The LPIII exceedance probability curve of the total runoff volumes of the wave belongs to annual maximum discharges of the Váh: Liptovský Mikuláš for part I. and part II. (1931–2015) were calculated by Log-Pearson type III. probability distribution. The exceedance curves of the total

runoff volumes are presented in (Figure 11). T -year annual runoff volumes V for selected parts are listed in Table 4. The average difference of the T -year total volumes V and T -year total volumes V_I and V_{II} belongs to annual maximum discharges for Q_{100} can be 22% and for Q_{1000} the difference is 43%.

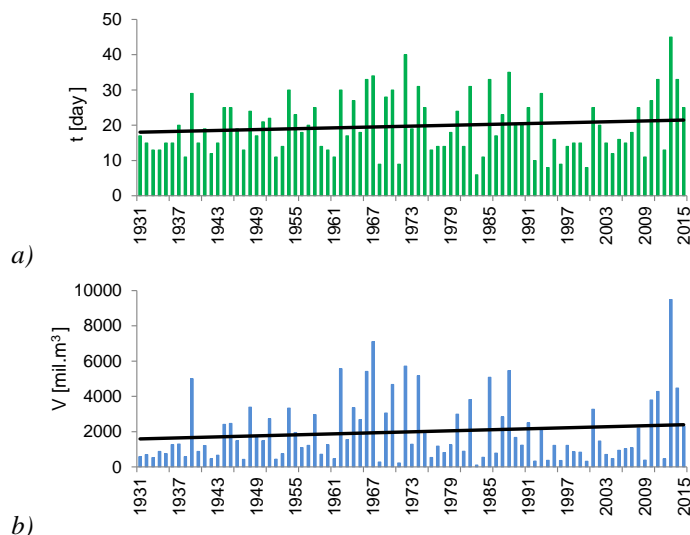


Fig. 8. Values of a) total duration of the waves annual maximum discharges, b) total runoff volumes of the wave belongs to annual maximum discharges, Váh: Liptovský Mikuláš (1931–2015).

Table 3. T -year maximum discharges Q_{max} [m^3s^{-1}] and T -year annual total runoff volumes V [mil. m^3] belongs to annual maximum discharges, Váh: Liptovský Mikuláš (1931–2015) (Log-Pearson III)

Váh: Liptovský Mikuláš (1931–2015)				
	Q_{50}	Q_{100}	Q_{500}	Q_{1000}
Q_T [m^3s^{-1}]	426	521	809	969
V [mil. m^3]	8633	10978	17790	21389

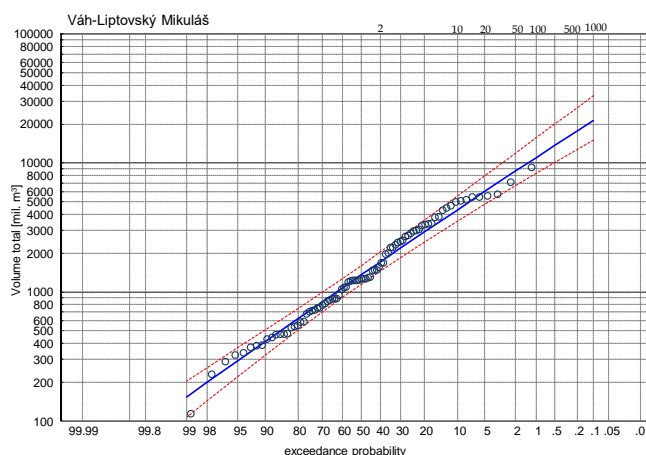


Fig. 9. The LPIII exceedance probability curve of the total runoff volume of the wave belongs to annual maximum discharges, Váh: Liptovský Mikuláš (1931–2015).

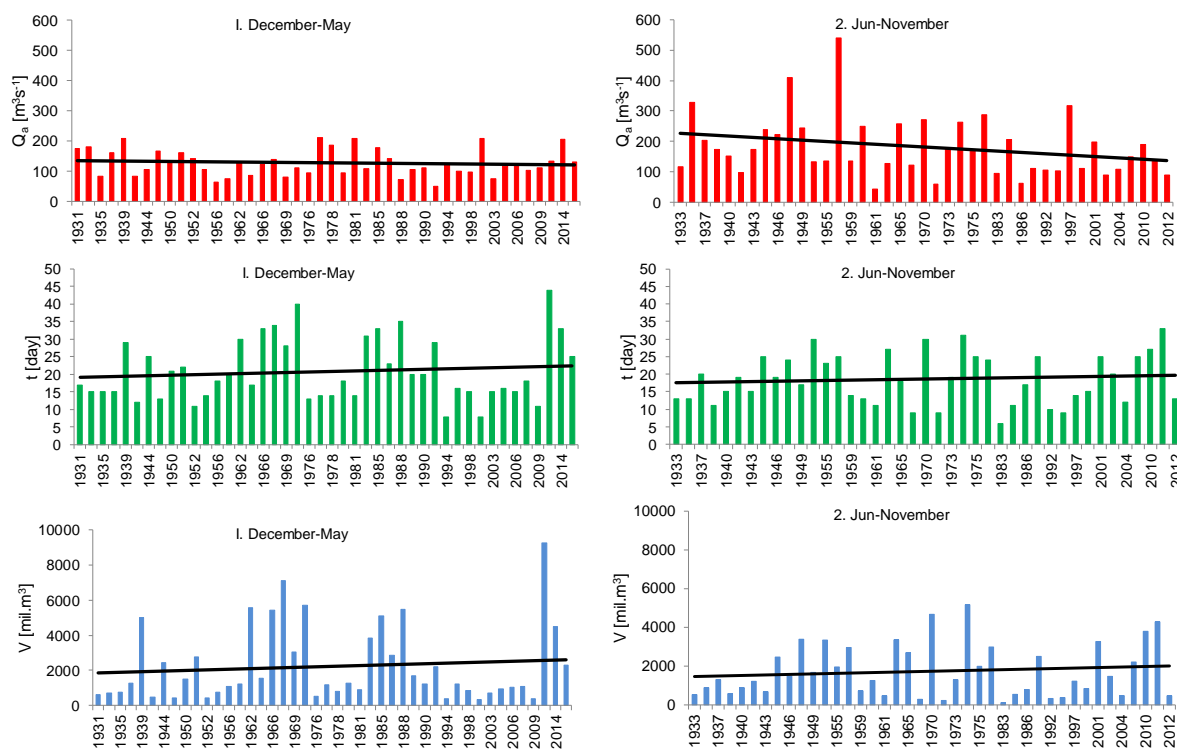


Fig. 10. Values of the annual maximum discharges, total duration of the wave belongs to annual maximum discharges and total wave volumes belongs to annual maximum discharges, Váh: Liptovský Mikuláš (1931–2015) (left: part I. and right: part II.).

Table 4. T-year annual total runoff volumes V_I [mil. m³] and V_{II} [mil. m³] for selected parts, Váh: Liptovský Mikuláš (1931–2015) (Log-Pearson III)

Váh: Liptovský Mikuláš (1931–2015)				
	V_{50}	V_{100}	V_{500}	V_{1000}
I. December–May				
V_I [mil. m ³]	10760	14411	26502	33707
II. June–November				
V_{II} [mil. m ³]	6981	8503	12434	14293

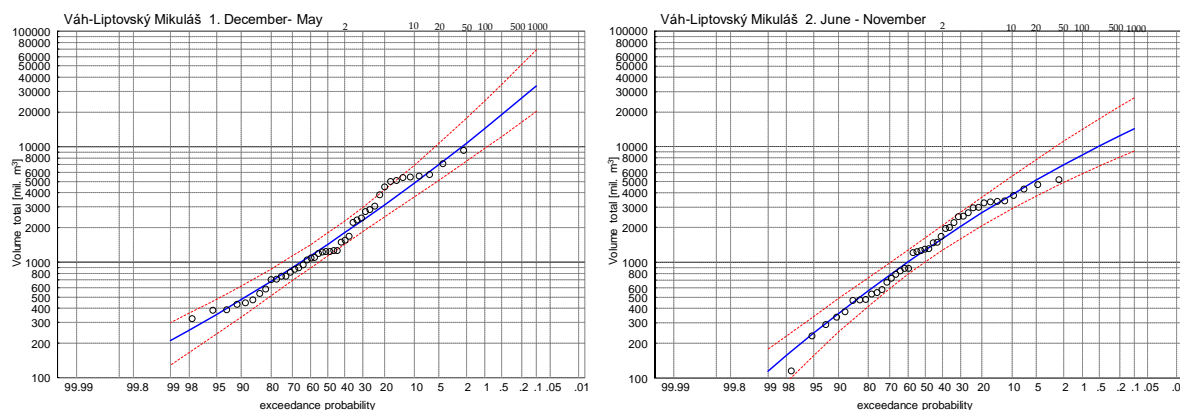


Fig. 11. The LPIII exceedance probability curve of the total runoff volumes of the wave belongs to annual maximum discharges, Váh: Liptovský Mikuláš (1931–2015) (left: part I and right: part II.).

Conclusion

Our analysis showed that in terms of the analyzed period 1931–2015, the maximum annual discharges have a decreasing linear trend and the annual maximum runoff volumes with a duration $t=2$ days have a slightly linear decreasing trend. Dividing the observed time period into two parts according to the occurrence of annual maximum discharges (I. December–May and II. June–November), the analysis showed a markedly decreasing trend at maximum annual discharges and a slightly decreasing trend in annual maximum runoff volumes with a duration $t=2$ days mainly in part II. June–November. On the contrary, the analysis of the total wave length and the total volume of wave belongs to annual maximum discharge showed an increasing linear trend in terms of the whole observed part 1931–2015. During the part I. December–May, the analysis showed a higher increasing trend of the total wave length and the total wave volume. The maximum annual discharges show only a slightly decreasing trend in the part I. December–May.

In conclusion, we can state that the given analysis showed on average decrease in annual maximum flows and also maximum annual volumes at $t=2$ days for the whole period. At the same time, an increase in the duration of the waves (according to our selected criteria) and the total volume of the waves was recorded. These changes are more pronounced especially for the part I. December–May. The snow melting in the mountain tributaries catchments and rainfall may cause such trend.

In addition to the analysis of volume changes, we focused on the use of one type of theoretical probability distribution – Log-Pearson distribution III. type. The LP III probability distribution showed a high sensitivity to the inclusion of extremes in the underlying data series. We can state that this distribution is suitable for maximum flows with a longer repetition time.

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